Time-Varying Filter Banks with Variable System Delay

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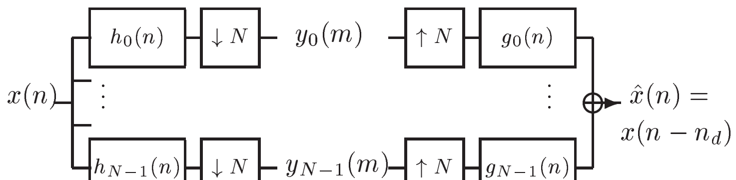
GOAL

Design method for modulated filter banks with perfect reconstruction and

- system delay independent of filter length
- time-varying filters
- time-varying number of bands



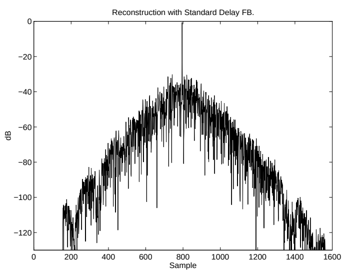
An N - channel filter bank with a system delay of n_d samples and filter length L.

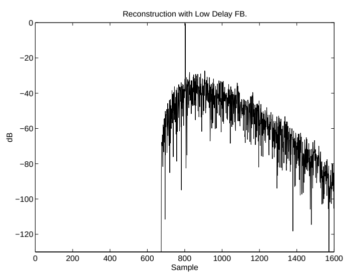
Standard system delay: $n_d = L - 1$

PRESENT STATE

Variable system delay, $n_d \geq N - 1$

- Improve the magnitude response for a given system delay
- Reduce the system delay for a given magnitude response e.g. for real time applications
- Shape quantization noise e.g. to eliminate "pre-echoes" in audio coding





Standard Delay　　Low Delay
Filter bank. Reconstructed impulse with quantization noise from filter banks with 128 bands.

PROBLEM

Make FB's with a variable system delay time-varying, to adapt to local signal statistics

APPROACH

Time-varying Polyphase Notation

$$\mathbf{Y}(z) = \mathbf{X}(z) \cdot \mathbf{P_a}(z, m)$$

$$\hat{\mathbf{X}}(z) = \mathbf{Y}(z) \cdot \mathbf{P_s}(z, m)$$

with time index m at lower sampling rate.

Rule for time varying systems:

$$\mathbf{P}(z, m) \cdot z^{-1} = z^{-1} \cdot \mathbf{P}(z, m-1)$$

Suitable decomposition of $\mathbf{P_a}(z, m)$, $\mathbf{P_s}(z, m)$ into simple matrices.
2 design variables (length, delay) $\Rightarrow$
2 types of "Filter Matrices" needed.

Filter Matrices with Bi-Diagonal form

Zero-Delay Matrices – increase the filter length by N, but not the system delay.

$$\mathbf{G}_i(z, m) := \begin{bmatrix} z^{-1} g_0^i(m) & & & & & 1 \\ & \ddots & & & \iddots & \\ & & z^{-1} g_{N/2-1}^i(m) & 1 & & \\ & & 1 & 0 & & \\ & \iddots & & & \ddots & \\ 1 & & & & & 0 \end{bmatrix}$$

$$\mathbf{G}_i^{-1}(z) = \begin{bmatrix} 0 & & & & & 1 \\ & \ddots & & & \iddots & \\ & & 0 & 1 & & \\ & & 1 & -z^{-1} g_{N/2-1}^i(m) & & \\ & \iddots & & & \ddots & \\ 1 & & & & & -z^{-1} g_0^i(m) \end{bmatrix}$$

Maximum-Delay Matrices – increase the filter length by N, the system delay by $2N$.

$$\mathbf{B}_i(z, m) := \begin{bmatrix} b_0^i(m) & & & & & z^{-1} \\ & \ddots & & & \iddots & \\ & & b_{N/2-1}^i(m) & z^{-1} & & \\ & & z^{-1} & 0 & & \\ & \iddots & & & \ddots & \\ z^{-1} & & & & & 0 \end{bmatrix}$$

$$\mathbf{B}_i^{-1}(z,m)\cdot z^{-2} =$$

$$= \begin{bmatrix} 0 & & & & & z^{-1} \\ & \ddots & & & \iddots & \\ & & 0 & z^{-1} & & \\ & & z^{-1} & -b^i_{N/2-1}(m-1) & & \\ & \iddots & & & \ddots & \\ z^{-1} & & & & & -b^i_0(m-1) \end{bmatrix}$$

Additional matrices

Shift Matrix - for fine tuning of system delay, reduces delay by 1 sample.

$$\mathbf{S}(z) := \begin{bmatrix} 0 & 0 & \cdots & 0 & z \\ 1 & 0 & \cdots & & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

Transform Matrices - e.g a DCT IV, for modulation

$$\mathbf{T_a}(m)\ ,\ \mathbf{T_s}(m)$$

Cascade them to design polyphase matrices for the desired filter length and system delay

$$\mathbf{P_a}(z,m) = \mathbf{S}^{n_a}(z)\cdot\prod_{i=0}^{\mu-1}\mathbf{B}_i(z,m)\cdot\prod_{j=1}^{\nu}\mathbf{G}_j(z,m)\cdot\mathbf{T_a}(m)$$

$$\mathbf{P_s}(z,m) = \mathbf{P_a}^{-1}(z,m)z^{-d}\cdot\mathbf{S}^{n_t}(z) =$$

$$= \mathbf{T_s}(m)\cdot\prod_{j=\nu}^{1}\mathbf{G}_j^{-1}(z,m)\cdot\prod_{i=\mu-1}^{1}\left(\mathbf{B}_i^{-1}(z,m-2(\mu-1-i))z^{-2}\right)\cdot\mathbf{S}^{n_s}(z)$$

where $\mathbf{T_a}(m)\cdot\mathbf{T_s}(m) = \mathbf{I}$.

Switching the Number of Bands

Use N_1 band structure for N_2 band FB. $(N_1 > N_2)$. Let $\mathbf{X}(z)$ be such that

$$\mathbf{X}(z)\cdot\mathbf{S}^{n_a}(z) =$$

$$[X'_0(z),\ldots,X'_{N_2/2-1}(z),\underbrace{0,\ldots,0}_{N_1-N_2},X'_{N_2/2}(z),\ldots,X'_{N_2-1}(z)]$$

$$\mathbf{T_a}(m) = \begin{bmatrix} \mathbf{T}_u \\ \hline \mathbf{0} \\ \hline \mathbf{T}_d \end{bmatrix}\ ,\ \ \mathbf{T_s}(m) = \left[\ \mathbf{T}_l\ \middle|\ \mathbf{0}\ \middle|\ \mathbf{T}_r\ \right]$$

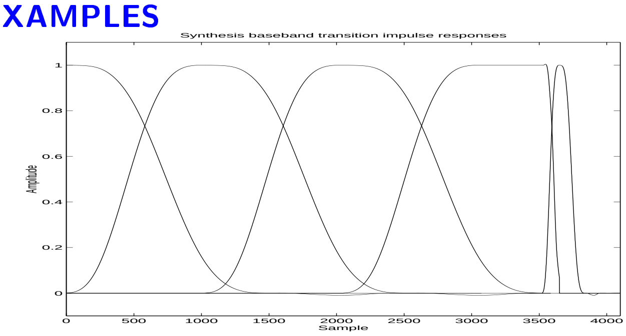
Their product is the $N_1\times N_1$ matrix

$$\mathbf{T_a}(m)\cdot\mathbf{T_s}(m) = \left[\begin{array}{c|c|c} \mathbf{I} & & \mathbf{0} \\ & \mathbf{0} & \\ \mathbf{0} & & \mathbf{I} \end{array}\right]$$

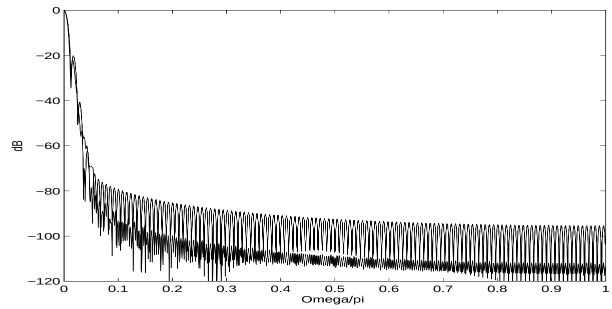
Optimization of Matrix Coefficients

- Is used to obtain the desired frequency responses
- A specialized version of the method of "conjugate directions" was developed
- Converges even for big filter banks

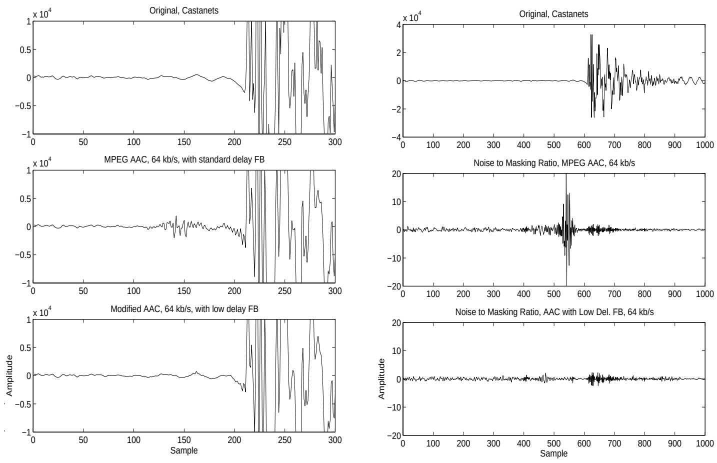
EXAMPLES



Synthesis baseband impulse responses, switch from 1024 bands, 4096 taps, 2047 delay (4 times overlap), to 128 bands 512 taps, 255 delay.



Magnitude response of the 128 band low delay prototype vs. standard delay (upper line), both 255 delay.



Signal Noise to Masking Ratio

Center: pre-echo of MPEG 2 Advanced Audio Coding (AAC) Coder, original FB with standard delay 255 samples in 128 band mode,
bottom: eliminated pre-echo of modified AAC coder, with Low Delay 191 samples.

RESULTS

A design method for

- Arbitrary filter length
- Arbitrary delay $n_d \geq N-1$
- Switch between any number of bands $N \geq 0$