A General Formulation for Modulated Perfect Reconstruction Filter Banks with Variable System Delay

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Abstract

This paper introduces a new formulation for analysis and design of modulated filter banks. A unique feature of the formulation is that it provides explicit control of the input-to-output system delay. The paper discusses minimum delay filter banks and demonstrates that truly exact reconstruction is possible in this context.

The formulation provides a broad range of design flexibility within a compact framework and allows for the design of a variety of computationally efficient modulated filter banks with different numbers of bands and virtually arbitrary lengths.

1 Introduction

Subband analysis/synthesis filter banks have been a topic of vigorous study for many years now. In the most common mode of operation, an analysis filter bank first splits the input signal into several frequency bands and then decimates each subband to its Nyquist sampling rate. The synthesis filter bank performs the dual operation by upsampling the subbands, and then filtering them to remove the imaged spectral copies. These outputs are summed to produce the reconstructed input.

Modulated filter banks are those in which a baseband filter is implicitly moduated, via a transform (such as a DCT), to create the bank. These filter banks are typically very efficient computationally because fast transforms algorithms are generally employed. There have been many contributions in the literature focusing on various aspects of the design problem [1], [2], [5], [6], [7].

In this paper, a new formulation is introduced that attempts to provide a broader range of design flexibility than reported previously while maintaining a simple and compact framework. The new formulation allows for the design of a variety of efficient modulated filter banks with different numbers of bands and virtually arbitrary lengths. It also allows for the simultaneous control over the overall system delay for a given filter length.

2 The Basic Matrix Framework

The starting point of the discussion given here is the simple N-band, $2N$-length, cosine modulated filter bank of the form

$$y_k(m) = \sum_{n=0}^{2N-1} x(mN+n)h(n)\cos(\frac{\pi}{N}(k+0.5)(n+0.5-\frac{N}{2})) \tag{1}$$

for all integer m, where $y_k(m)$ is the output of the k'th subband channel at the m'th interval. As a side note, we point out that this particular filter bank is essentially the TDAC filter bank proposed in [1]. The filter bank may be viewed as processing the input in blocks. For every block of N samples, where m may be viewed as the block index for $x(mN+n)$, N output samples in

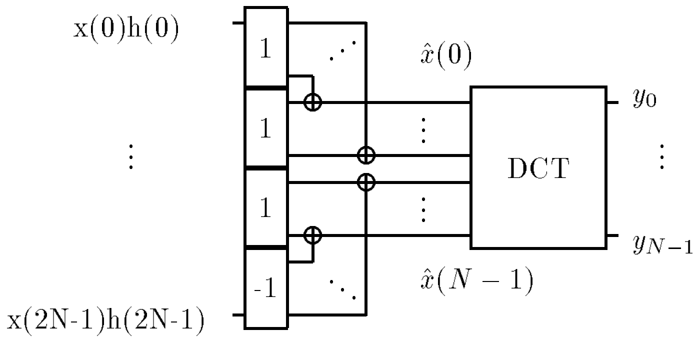

Figure 1: The analysis filter bank for the N-band, $2N$-length, modulated filter bank

the variable k are produced: $y_k(m) \quad k = 0, 1, \ldots, N-1$. For convenience and without loss of generality, we can perform the analysis over the m'th interval, and thus can simplify (1) to

$$y_k = \sum_{n=0}^{2N-1} x(n)h(n)\cos(\frac{\pi}{N}(k+0.5)(n+0.5-\frac{N}{2})) \tag{2}$$

Exploiting the symmetries embodied in the identities

$$\cos(\frac{\pi}{N}(k+0.5)((N+n)+0.5)) = -\cos(\frac{\pi}{N}(k+0.5)((N-1-n)+0.5))$$

and

$$\cos(\frac{\pi}{N}(k+0.5)((-n)+0.5)) = \cos(\frac{\pi}{N}(k+0.5)((n-1)+0.5))$$

the analysis equation can be rewritten:

$$\begin{aligned} y_k = & \sum_{n=0}^{N/2-1} x(\frac{N}{2}-1-n)h(\frac{N}{2}-1-n)\cos(\frac{\pi}{N}(k+0.5)(n+0.5)) \\ & + \sum_{n=0}^{N-1} x(n+\frac{N}{2})h(n+\frac{N}{2})\cos(\frac{\pi}{N}(k+0.5)(n+0.5)) \\ & - \sum_{n=0}^{N/2-1} x(2N-1-n)h(2N-1-n)\cos(\frac{\pi}{N}(k+0.5)(\frac{N}{2}+n+0.5)) \end{aligned}$$

A close examination of these symmetries shows that the analysis can be written as a type of "folding" operation followed by a cosine transform. This is illustrated in Figure 1, where the boxes symbolize multiplications with $+1$ or -1 respectively. With $\hat{x}(n)$ as defined in the picture, $y_k(m)$ has the form

$$y_k = \sum_{n=0}^{N-1} \hat{x}(n)\cos(\frac{\pi}{N}(k+0.5)(n+0.5))$$

This is the DCT of $\hat{x}$ with odd spaced frequencies [4]. In reference to Figure 1, the signal and system components can now be written in matrix form with z-domain matrices and vectors. Let the input be represented by the time domain vector, $\mathbf{x} = [x(0), \ldots, x(N-1)]$ and the z-domain vector

$$\mathbf{X} = [X_0(z), \ldots, X_{N-1}(z)].$$

The input and output vectors of the DCT can be written as

$$\hat{\mathbf{X}} = [\hat{X}_0(z), \ldots, \hat{X}_{N-1}(z)] \quad \text{and} \quad \mathbf{Y} = [Y_0(z), \ldots, Y_{N-1}(z)]$$

respectively. The DCT transform matrix is $\mathbf{T}$, with elements

$$\mathbf{t}(n,k) = \cos(\frac{\pi}{N}(k+0.5)(n+0.5)) \quad n,k = 0, \ldots, N-1$$

and a "folding" matrix $\mathbf{F_a}$, which converts $\mathbf{X}$ into $\hat{\mathbf{X}}$:

$$\mathbf{F_a} = \begin{bmatrix} 0 & & h(0)z^{-1} & h(N) & & 0 \\ & \iddots & & & \ddots & \\ h(N/2-1)z^{-1} & & 0 & & & h(N+N/2-1) \\ h(N/2)z^{-1} & & & & & -h(N+N/2) \\ & \ddots & & & \iddots & \\ 0 & & h(N-1)z^{-1} & -h(2N-1) & & 0 \end{bmatrix}$$

Thus $\hat{\mathbf{X}}$ can be written as $\hat{\mathbf{X}} = \mathbf{X} \cdot \mathbf{F_a}$. We can further decompose $\mathbf{F_a}$ into the matrices $\mathbf{D}$ and $\mathbf{F}$ where

$$\mathbf{D} = \begin{bmatrix} z^{-1} & & & & \\ & \ddots & & & \\ & & z^{-1} & & 0 \\ 0 & & & 1 & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix} \tag{3}$$

$$\mathbf{F} = \begin{bmatrix} 0 & & h(0) & h(N) & & 0 \\ & \iddots & & & \ddots & \\ h(N/2-1) & & 0 & & & h(N+N/2-1) \\ h(N/2) & & & & & -h(N+N/2) \\ & \ddots & & & \iddots & \\ 0 & & h(N-1) & -h(2N-1) & & 0 \end{bmatrix}$$

which results in $\hat{\mathbf{X}} = \mathbf{X} \cdot \mathbf{F} \cdot \mathbf{D}$ and $\mathbf{Y} = \mathbf{X} \cdot \mathbf{F} \cdot \mathbf{D} \cdot \mathbf{T}$. This scheme also lends itself to a fast implementation, because most of the computations required are for the DCT, which can be done with fast algorithms. The other matrices only contain 2N multiplications (with the window function) and N summations. It is easy to see that the synthesis consists of the inverse operations: $\mathbf{X} = \mathbf{Y} \cdot \mathbf{T^{-1}} \cdot \mathbf{D^{-1}} \cdot \mathbf{F^{-1}}$. For this simple filter bank, the inverses are very easy to compute:

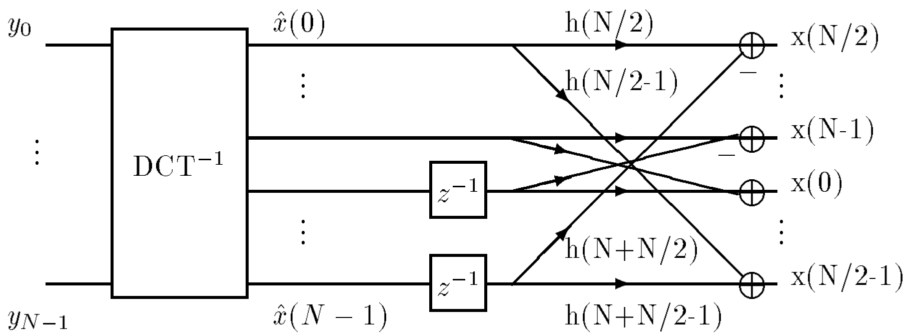

Figure 2: The corresponding synthesis flow graph for the N-band, $2N$-length modulated filter bank

$\mathbf{T^{-1}} = \mathbf{T} \cdot \frac{2}{N}$, which is the inverse DCT; and the inverse of the delay matrix $\mathbf{D}$ is

$$\mathbf{D^{-1}} \cdot z^{-1} = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & & z^{-1} & & \\ & & & & \ddots & \\ & & & & & z^{-1} \end{bmatrix}.$$

The multiplication with z^{-1} is to make the filter bank causal. If the window function $h(n)$ has the symmetries required for the TDAC filterbank (which are not necessary here), then $\mathbf{F^{-1}} = \mathbf{F}^T$. With these component inverses, the synthesis becomes

$$z^{-1} \cdot \mathbf{X} = \mathbf{Y} \cdot \mathbf{T} \cdot \frac{2}{N} \cdot (z^{-1} \cdot \mathbf{D^{-1}}) \cdot \mathbf{F}^{-1}$$

These matrix operations lead to the structure shown in Figure 2—a structure which was first introduced by Malvar. Similarly, the analysis filter bank can also be drawn in this way, with butterflies.

This discussion illustrates the new matrix formulation for the simple case of the N-band $2N$-length modulated filter bank. This mathematical framework is next extended to include a wide variety of filter banks, with different window functions, different lengths, and with other transforms.

3 Extending the Matrix Framework

The generalization of the formulation is illustrated here for the class of cosine modulation functions. First consider filter banks with odd DCTs of the form $\cos(\frac{\pi}{N}(k+0.5)(n-0.5+\frac{N}{2}))$ and filter lengths $2LN-1$ where L is a positive integer. The analysis equation (for the m'th interval) is then

$$y_k = \sum_{n=0}^{2LN-1} x(n)h(n)\cos(\frac{\pi}{N}(k+0.5)(n+0.5-\frac{N}{2})).$$

Exploiting the cosine symmetry

$$\cos(\frac{\pi}{N}(k+0.5)((n+2N)+0.5)) = -\cos(\frac{\pi}{N}(k+0.5)(n+0.5)),$$

a folding matrix for longer windows with lengths $2LN$ can be written. The elements of this matrix have a diamond shaped pattern:

$$\mathbf{F_a} = \begin{bmatrix} & & p(0)z^{-1} & p(N) & & \\ & \iddots & & & \ddots & \\ p(N/2-1)z^{-1} & & & & & p(N+N/2-1) \\ p(N/2)z^{-1} & & & & & -p(N+N/2) \\ & \ddots & & & \iddots & \\ & & p(N-1)z^{-1} & -p(2N-1) & & \end{bmatrix}$$

where $p(n) = \sum_{m=0}^{L-1} h(n+m2N)(-1)^m z^{-2(L-1-m)}$ and $h(n)$ is a window function. For the synthesis filter bank

$$\mathbf{F_s} = \begin{bmatrix} & & p(N/2-1) & p(N/2) & & \\ & \iddots & & & \ddots & \\ p(0) & & & & & p(N-1) \\ p(N)z^{-1} & & & & & -p(2N-1)z^{-1} \\ & \ddots & & & \iddots & \\ & & p(N+N/2-1)z^{-1} & -p(N+N/2)z^{-1} & & \end{bmatrix}$$

with $p(n) = \sum_{m=0}^{\infty} h_s(n+m2N)(-1)^m z^{-2m}$ and where $h_s(n)$ is the synthesis window. The indexing of the sum goes to infinity because the synthesis filter bank is IIR.

Now consider N-band, $(2LN-1)$-length filter banks based on modulating cosine functions of the form $\cos(\frac{\pi}{N}(k+0.5)(n-0.5))$. The analysis equation is then

$$y_k = \sum_{n=0}^{2LN-1} x(n)h(n)\cos(\frac{\pi}{N}(k+0.5)(n+0.5))$$

and the elements of the analysis folding matrix take on a cross-shaped appearance:

$$\mathbf{F_a} = \begin{bmatrix} p(0)z^{-1} & & & & & -p(N) \\ & \ddots & & & \iddots & \\ & & p(N/2-1)z^{-1} & -p(N+N/2-1) & & \\ & & -p(N+N/2) & p(N/2)z^{-1} & & \\ & \iddots & & & \ddots & \\ -p(2N-1) & & & & & p(N-1)z^{-1} \end{bmatrix}$$

with $p(n) = \sum_{m=0}^{L-1} h(n+m2N)(-1)^m z^{-2(L-1-m)}$. The synthesis folding matrix also has the same appearance:

$$\mathbf{F_s} = \begin{bmatrix} p(0) & & & & & -p(2N-1)z^{-1} \\ & \ddots & & & \iddots & \\ & & p(N/2-1) & -p(N+N/2)z^{-1} & & \\ & & -p(N+N/2-1)z^{-1} & p(N/2) & & \\ & \iddots & & & \ddots & \\ -p(N)z^{-1} & & & & & p(N-1) \end{bmatrix}$$

Many other types of filter banks are possible as well. For example, modulating functions of the form $\cos(\frac{\pi}{N}(k+0.5)(n+0.5-N))$ are possible and will also result in cross-shaped synthesis folding matrices, but with the z^{-1} on the main diagonal (which is needed e.g. for the minimum delay filter banks in section 3.1). Similarly modulating functions of the form $\cos(\frac{\pi}{N}k(n+0.5))$ also result in cross-shaped analysis-synthesis folding matrices, while functions of the form $\cos(\frac{\pi}{N}k(n-0.5+\frac{N}{2}))$ result in diamond-shaped folding matrices.

To construct the synthesis filter bank which makes perfect reconstruction possible, it is necessary to find the inverse of the folding matrices. For the important cases of the diamond-shaped and the cross-shaped matrices as above, a general inverse can be computed.

Consider the diamond-shaped matrix

$$\mathbf{F_d} = \begin{bmatrix} & & a_0 & b_0 & & \\ & \iddots & & & \ddots & \\ a_{N/2-1} & & & & & b_{N/2-1} \\ a_{N/2} & & & & & b_{N/2} \\ & \ddots & & & \iddots & \\ & & a_{N-1} & b_{N-1} & & \end{bmatrix}$$

where a_i and b_i are polynomials in z or ratios of polynomials. It can be shown (with sub-determinants), that the inverse is

$$\mathbf{F_d^{-1}} = \begin{bmatrix} & & a'_{N/2-1} & a'_{N/2} & & \\ & \iddots & & & \ddots & \\ a'_0 & & & & & a'_{N-1} \\ b'_0 & & & & & b'_{N-1} \\ & \ddots & & & \iddots & \\ & & b'_{N/2-1} & b'_{N/2} & & \end{bmatrix}$$

with

$$a'_i = \frac{b_{N-1-i}}{a_i b_{N-1-i} - b_i a_{N-1-i}} \qquad b'_i = \frac{-a_{N-1-i}}{a_i b_{N-1-i} - b_i a_{N-1-i}}$$

and $i = 0 \ldots N-1$. Similarly for a cross-shaped matrix $\mathbf{F_c}$:

$$\mathbf{F_c} = \begin{bmatrix} a_0 & & & & & b_0 \\ & \ddots & & & \iddots & \\ & & a_{N/2-1} & b_{N/2-1} & & \\ & & b_{N/2} & a_{N/2} & & \\ & \iddots & & & \ddots & \\ b_{N-1} & & & & & a_{N-1} \end{bmatrix}$$

the inverse is $\mathbf{F_c^{-1}}$:

$$\mathbf{F_c^{-1}} = \begin{bmatrix} a'_0 & & & & & b'_0 \\ & \ddots & & & \iddots & \\ & & a'_{N/2-1} & b'_{N/2-1} & & \\ & & b'_{N/2} & a'_{N/2} & & \\ & \iddots & & & \ddots & \\ b'_{N-1} & & & & & a'_{N-1} \end{bmatrix}$$

with

$$a'_i = \frac{a_{N-1-i}}{a_i a_{N-1-i} - b_i b_{N-1-i}} \qquad b'_i = \frac{-b_i}{a_i a_{N-1-i} - b_i b_{N-1-i}}$$

and $i = 0, \ldots, N-1$.

With these inverses it is now possible to construct the synthesis filter bank for any analysis window function and any modulating function which leads to a diamond- or cross-shaped folding matrix, as long as it is invertible. Conversely it is possible to construct an analysis filter bank from a given synthesis filter bank given the same conditions. Most often, one wishes to have both analysis and synthesis filter banks be FIR. To accommodate this specification, the elements of the folding matrices may be designed iteratively with the constraint that the inverse matrices are FIR. The addition of this constraint does not represent a practical problem for reasonable filter lengths of interest in real world applications.

3.1 Controlling the System Delay

This formulation of the analysis/synthesis problem is constrained structurally to guarantee exact reconstruction. It has the additional advantage that its structure can be arranged to guarantee a prespecified input-to-output system delay. Observe that if the folding matrix can be written as a product of delay matrices, ($\mathbf{D}$) of the form shown in (3), and coefficient matrices with real or complex coefficients, the matrix inverses are always FIR, which means both, analysis and synthesis filter bank, will be FIR. This can be done by using coefficient matrices $\mathbf{F}$ and $\mathbf{C}_i$, which have the form

$$\mathbf{F} = \begin{bmatrix} & & d_0 & d_N & & \\ & \iddots & & & \ddots & \\ d_{N/2-1} & & & & & d_{N+N/2-1} \\ d_{N/2} & & & & & d_{N+N/2} \\ & \ddots & & & \iddots & \\ & & d_{N-1} & d_{2N-1} & & \end{bmatrix}$$

$$\mathbf{C}_i = \begin{bmatrix} c_0^i & & & & & c_N^i \\ & \ddots & & & \iddots & \\ & & c_{N/2-1}^i & c_{N+N/2-1}^i & & \\ & & c_{N/2}^i & c_{N+N/2}^i & & \\ & \iddots & & & \ddots & \\ c_{N-1}^i & & & & & c_{2N-1}^i \end{bmatrix}$$

where $d_0 \ldots d_{2N-1}$ and $c_0^i \ldots c_{2N-1}^i$ are the elements of $\mathbf{F}$ and $\mathbf{C}_i$ respectively. It can be shown that valid folding matrices are achieved if the analysis filter banks have the form

$$\mathbf{F_a} = (\prod_{i=1}^{m} \mathbf{C}_i \cdot \mathbf{D}_i^2) \cdot \mathbf{F} \cdot \mathbf{D} \tag{4}$$

and the synthesis banks have the form

$$\mathbf{F_s} = \mathbf{D}^{-1} \cdot \mathbf{z}^{-1} \cdot \mathbf{F}^{-1} \cdot (\prod_{i=0}^{m-1} \mathbf{D}_{m-i}^{-2} \cdot \mathbf{z}^{-2} \cdot \mathbf{C}_{m-i}^{-1}). \tag{5}$$

The resulting filter bank has length $K = 2Nm + 2N$ and a delay of $K - 1$ samples, which is the typical system delay for filter banks.

The delay in above section (except for the transform block delay) results from the multiplication by z^{-1} to make the filter bank causal. To facilitate systems with other delays, a folding matrix is needed with a form such that its inverse has no positive powers of z. As can be seen from the general inverse for the cross-shaped folding matrix, this can be achieved with the following two matrices, **E** and **G**:

$$\mathbf{E}_i = \begin{bmatrix} 0 & & & & & e^i_N \\ & \ddots & & & \iddots & \\ & & 0 & e^i_{N+N/2-1} & & \\ & & e^i_{N+N/2} & e^i_{N/2}z^{-1} & & \\ & \iddots & & & \ddots & \\ e^i_{2N-1} & & & & & e^i_{N-1}z^{-1} \end{bmatrix}$$

where e^i_j is real or complex. The inverse is

$$\mathbf{E}_i^{-1} = \begin{bmatrix} \hat{e}^i_0 z^{-1} & & & & & \hat{e}^i_N \\ & \ddots & & & \iddots & \\ & & \hat{e}^i_{N/2-1}z^{-1} & \hat{e}^i_{N+N/2-1} & & \\ & & \hat{e}^i_{N+N/2} & 0 & & \\ & \iddots & & & \ddots & \\ \hat{e}^i_{2N-1} & & & & & 0 \end{bmatrix}$$

with

$$\hat{e}^i_j = \frac{e_{N-1-j}}{-e_{N+j}e_{2N-1-j}}, \quad j = 0 \ldots N/2-1, \quad \text{and} \quad \hat{e}^i_{N+j} = \frac{e_{N+j}}{e_{N+j}e_{2N-1-j}}, \quad j = 0 \ldots N-1.$$

The second matrix is

$$\mathbf{G}_i = \begin{bmatrix} g^i_0 z^{-1} & & & & & g^i_N \\ & \ddots & & & \iddots & \\ & & g^i_{N/2-1}z^{-1} & g^i_{N+N/2-1} & & \\ & & g^i_{N+N/2} & 0 & & \\ & \iddots & & & \ddots & \\ g^i_{2N-1} & & & & & 0 \end{bmatrix}$$

with inverse

$$\mathbf{G}_i^{-1} = \begin{bmatrix} 0 & & & & & \hat{g}^i_N \\ & \ddots & & & \iddots & \\ & & 0 & \hat{g}^i_{N+N/2-1} & & \\ & & \hat{g}^i_{N+N/2} & \hat{g}^i_{N/2}z^{-1} & & \\ & \iddots & & & \ddots & \\ \hat{g}^i_{2N-1} & & & & & \hat{g}^i_{N-1}z^{-1} \end{bmatrix}$$

where

$$\hat{g}_j^i = \frac{e^i_{N-1-j}}{-e^i_{N+j}e^i_{2N-1-j}}, \quad j = N/2...N-1 \quad \text{and} \quad \hat{g}^i_{N+j} = \frac{e_{N+j}}{e^i_{N+j}e^i_{2N-1-j}}, \quad j = 0...N-1.$$

A product of the $\mathbf{E}_i$ matrices yields valid folding matrices and can support filters with good magnitude response characteristics. The analysis folding matrix is of the form

$$\mathbf{F_a} = \prod_{i=1}^{m} \mathbf{E}_i$$

and the synthesis matrix is the inverse,

$$\mathbf{F_s} = \prod_{i=0}^{m-1} \mathbf{E}^{-1}_{m-i}$$

The resulting length of the impulse response of the analysis and synthesis filter bank is $K = mN + 0.5N$, (where $m > 0$). The delay that is left is the transform block delay of $N-1$ samples, which is the minimum possible delay.

A combination of minimum delay and standard delay matrices yields folding matrices for low delay filter banks. Here minimum delay $\mathbf{G}_i$ matrices should be used in conjunction with the normal delay $\mathbf{C}_i$, $\mathbf{D}$, and $\mathbf{F}$ matrices to achieved the targeted system delay. The $\mathbf{G}_i$ matrices should be used here instead of the $\mathbf{E}_i$ matrices because their structure allows analysis-synthesis filters with good filter characteristics to be designed. The resulting form is

$$\mathbf{F_a} = (\prod_{i=1}^{m} \mathbf{C}_i \cdot \mathbf{D}_i^2) \cdot \mathbf{F} \cdot \mathbf{D} \cdot (\prod_{i=1}^{n} \mathbf{G}_i) \tag{6}$$

for the analysis filters and

$$\mathbf{F_s} = (\prod_{i=0}^{m-1} \mathbf{G}^{-1}_{m-i}) \cdot \mathbf{D}^{-1} \cdot z^{-1} \cdot \mathbf{F}^{-1} \cdot (\prod_{i=0}^{m-1} \mathbf{D}^{-2}_{m-i} \cdot z^{-2} \cdot \mathbf{C}^{-1}_{m-i}) \tag{7}$$

for the synthesis filters. The length of the impulse response is $K = m2N + nN + 2N$, and the delay is $m2N + 2N - 1$ samples. To illustrate this issue of low delay filter banks, examine Figure 3. It shows the magnitude responses for two baseband analysis filters. Both correspond to an 8 band filter bank. However, the one shown with the solid line is for an 8 tap system with a delay of 8 samples. The one shown by the dashed line corresponds to a 12-tap low delay systems with a delay of 8 samples. The improvement in quality that can be achieved for the same system delay is clearly visible. Moreover, the reconstruction is exact since exact reconstruction is guaranteed by the matrix structure. In conclusion, this matrix formulation provides a convenient design framework for constructing computationally efficient filter banks with a variety of different lengths and modulation kernels. Moreover, it allows the overall system delay to be specified by selecting the proper cascade of submatrices. The matrix parameters can be optimized iteratively to acheive good stopband and passband characteristics.

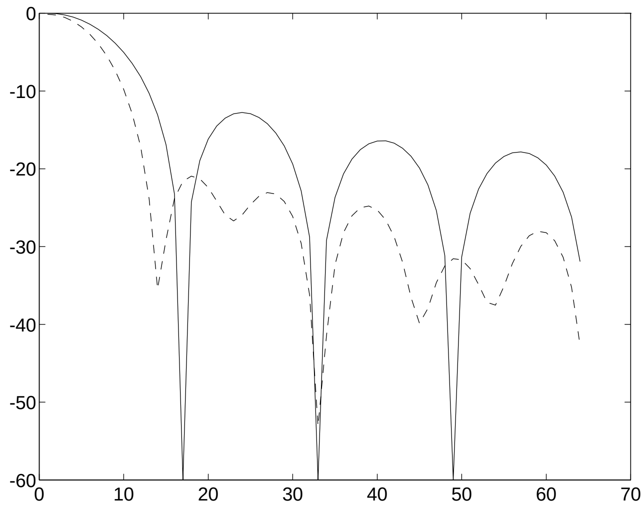

Figure 3: Magnitude response of two 8-band analysis filters. The solid line corresponds to a system with delay 8 and filter length 8. The dashed line corresponds to a system with delay 8 and filter length 12.

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