AN OVERVIEW OVER FILTER BANKS WITH LOW SYSTEM DELAY CAPABILITIES

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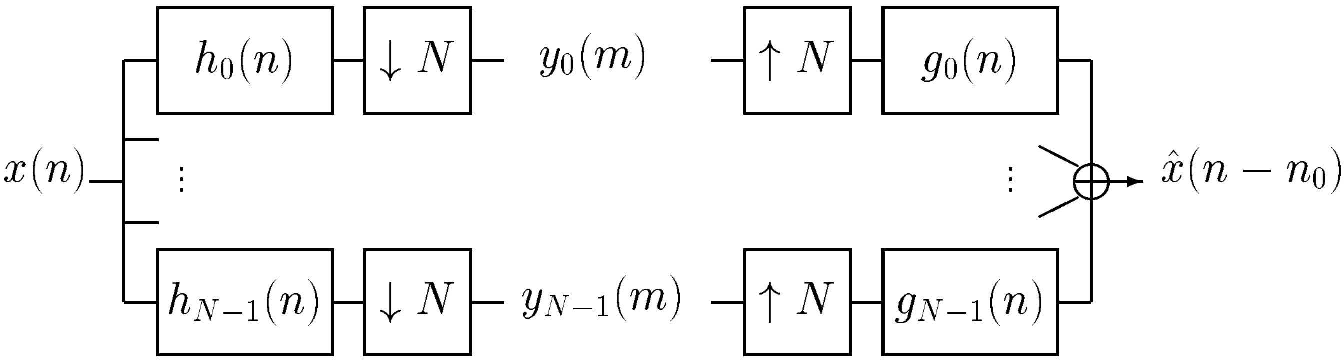
1 Introduction



An N - channel filter bank with critical downsampling and a system delay of n_0 samples.

- Perfect reconstruction: $\hat{x}(n) = x(n - n_0)$
- Near perfect reconstruction: $\hat{x}(n) \approx x(n - n_0)$
- Standard delay for length LN filters: $LN-1$ samples

2 Nayebi's Formulation in the Time Domain

(Nayebi,Barnwell,Smith, 87, 91 [6, 7, 8])

Define

$$\mathbf{P}_i = \begin{bmatrix} h_0(iN) & \cdots & h_0(iN+N-1) \\ & \vdots & \\ h_{N-1}(iN) & \cdots & h_{N-1}(iN+N-1) \end{bmatrix}$$

$$\mathbf{Q}_i = \begin{bmatrix} g_0(iN) & \cdots & g_0(iN+N-1) \\ & \vdots & \\ g_{N-1}(iN) & \cdots & g_{N-1}(iN+N-1) \end{bmatrix}$$

$i = 0, \ldots, L-1$

The reconstruction property leads to the equation system

$$
\begin{bmatrix}
\mathbf{P}_0^T & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{P}_1^T & \mathbf{P}_1^T & \mathbf{0} & \cdots & \mathbf{0} \\
\vdots & \vdots & \vdots & & \vdots \\
\mathbf{P}_{L-1}^T & \mathbf{P}_{L-2}^T & \mathbf{P}_{L-3}^T & \cdots & \mathbf{P}_0^T \\
\mathbf{0} & \mathbf{P}_{L-1}^T & \mathbf{P}_{L-2}^T & \cdots & \mathbf{P}_1^T \\
\vdots & \vdots & \vdots & & \vdots \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{P}_{L-1}^T
\end{bmatrix}
\begin{bmatrix}
\mathbf{Q}_0 \\ \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_{L-1}
\end{bmatrix}
=
\underbrace{\begin{bmatrix}
\mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{J}_N \\ \mathbf{0} \\ \vdots \\ \mathbf{0}
\end{bmatrix}}_{\mathbf{B}}
$$

$$
\mathbf{J}_N =
\begin{bmatrix}
0 & 0 & \cdots & 0 & 1 \\
0 & 0 & \cdots & 1 & 0 \\
\vdots & \vdots & & \vdots & \vdots \\
0 & 1 & \cdots & 0 & 0 \\
1 & 0 & \cdots & 0 & 0
\end{bmatrix}
$$

Optimization: Find $\mathbf{P}_i$ and $\mathbf{Q}_i$ which approximate the right hand side and the desired frequency responses.

For the standard system delay

$$
\mathbf{B} = [\underbrace{\mathbf{0}, \cdots, \mathbf{0}}_{L-1}, \mathbf{J}, \underbrace{\mathbf{0}, \cdots, \mathbf{0}}_{L-1}]^T
$$

For minimum system delay:

$$\mathbf{B} = [\mathbf{J}, \mathbf{0}, \cdots, \mathbf{0}]^T$$

Low system delay:

$$\mathbf{B} = [\mathbf{0}, \mathbf{J}, \mathbf{0}, \cdots, \mathbf{0}]^T$$

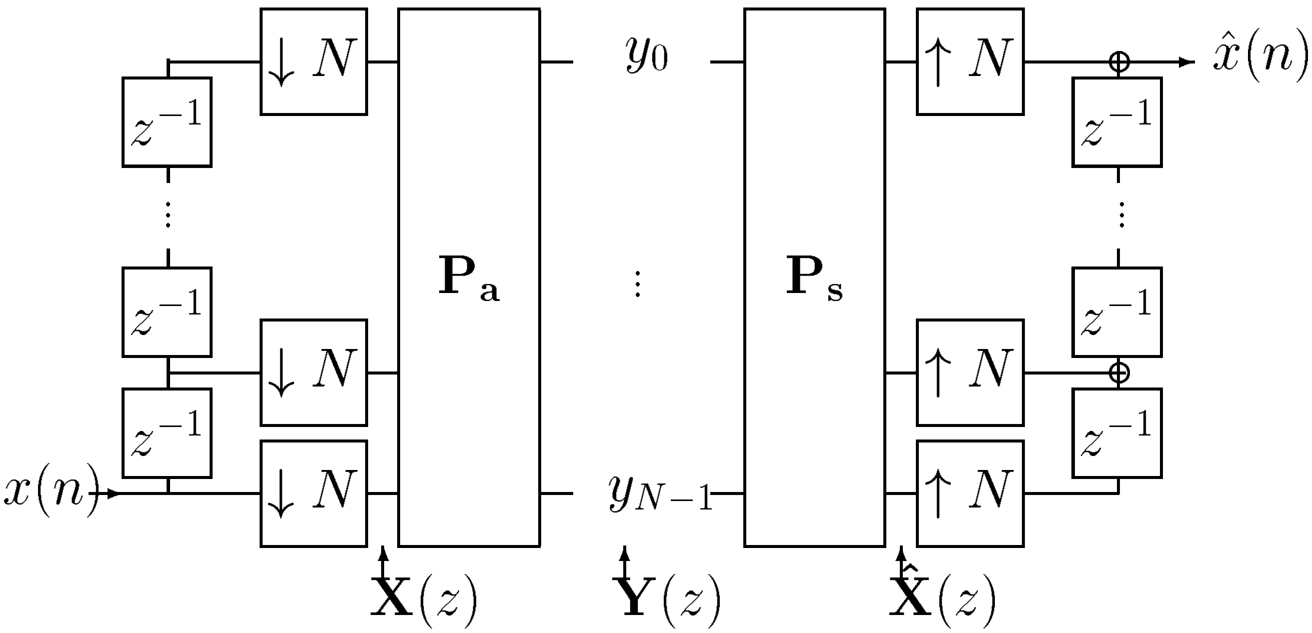
Properties

- Low system delay possible
- Very general approach
- But difficult optimization for big filter banks
- No perfect reconstruction

3 A New Formulation for Modulated Filter Banks

(Schuller, Smith, 94, [9, 10, 11, 12])

Polyphase representation of a filter bank



$\mathbf{P_a}(z)$ Analysis Polyphase Matrix

$\mathbf{P_s}(z)$ Synthesis Polyphase Matrix

$$\mathbf{P_a}(z) = \left[\, P_{n,k}(z) \,\right]_{n,k=0,\ldots,N-1}$$

$$\text{where} \quad P_{n,k}(z) = \sum_{m=0}^{L-1} h_k(n+2m) z^{-(L-1-m)}.$$

$$\mathbf{P_s}(z) = \left[\, P'_{k,n}(z) \,\right]_{n,k=0,\ldots,N-1}$$

$$\text{where} \;\; P'_{k,n}(z) = \sum_{m=0}^{L-1} g_k(n+2m)z^{-m}.$$

The analysis filtering and downsampling operation is

$$\mathbf{Y}(z) = \mathbf{X}(z) \cdot \mathbf{P_a}(z)$$

The upsampling and synthesis filtering operation is

$$\hat{\mathbf{X}}(z) = \mathbf{Y}(z) \cdot \mathbf{P_s}(z)$$

Synthesis for perfect reconstruction

$$\mathbf{P_s}(z) = \mathbf{P_a}^{-1}(z) \cdot z^{-d}$$

z^{-d} is introduced to make $\mathbf{P_s}(z)$ causal.

System Delay: $d \cdot N$ plus blocking delay, i.e.

$$n_0 = d \cdot N + N - 1$$

Modulated Filter Banks

- They are determined by the baseband filters
- Have an efficient implementation

Assume a modulated analysis filter bank like

$$h_k(n) = h(n) \cdot \cos\left(\frac{\pi}{N}(k+0.5)(n+0.5+n_0)\right)$$

$k = 0, \ldots, N-1,$

$n = 0, \ldots, LN-1$

and a Discrete Cosine Transformation $\mathbf{T}$ type IV

$$t(n,k) = \cos(\frac{\pi}{N}(k+0.5)(n+0.5)) \quad , \quad 0 \le n,k < N$$

For $n_0 = -N/2$ observe that

$$\mathbf{F_a}(z) := \mathbf{P_a}(z) \cdot \mathbf{T}^{-1} =$$

$$\begin{bmatrix} 0 & & P_0(z)z^{-1} & P_N(z) & & 0 \\ & \iddots & & & \ddots & \\ P_{N/2-1}(z)z^{-1} & & 0 & & & P_{N+N/2-1}(z) \\ P_{N/2}(z)z^{-1} & & & & & -P_{N+N/2}(z) \\ & \ddots & & & \iddots & \\ 0 & & P_{N-1}(z)z^{-1} & -P_{2N-1}(z) & & 0 \end{bmatrix}$$

This is a **Filter Matrix** with a *diamond* structure, where

$$P_k(z) = \sum_{m=0}^{L-1} h(m2N+k)(-1)^m z^{-2(L-1-m)}. \quad (1)$$

For the synthesis,

$$\mathbf{F_s}(z) = \mathbf{T} \cdot \mathbf{P_s}(z)$$

$\mathbf{F_s}(z)$ has the same structure as $\mathbf{F_a}(z)$

For $n_0 = 0$,

$$\mathbf{F_a} = \begin{bmatrix} P_0(z)z^{-1} & 0 & & & & -P_N(z) \\ & \ddots & & & \cdots & \\ & & P_{N/2-1}(z)z^{-1} & -P_{N+N/2-1}(z) & & 0 \\ 0 & & -P_{N+N/2}(z) & P_{N/2}(z)z^{-1} & & \\ & \cdots & & & \ddots & \\ -P_{2N-1}(z) & & 0 & & & P_{N-1}(z)z^{-1} \end{bmatrix}$$

with $P_k(z)$ as defined in equation (1). This is a Filter Matrix with a *bi-diagonal structure*

This is the **First step**: Decompose the polyphase matrices as

$$\mathbf{P_a}(z) = \mathbf{F_a}(z) \cdot \mathbf{T}$$

$$\mathbf{P_s}(z) = \mathbf{T}^{-1} \cdot \mathbf{F_s}(z)$$

Second Step: Further decompose the Filter Matrix $\mathbf{F_a}(z)$.

$\mathbf{F_a}(z)$ is chosen to be a cascade of the following matrices. Their properties determine the properties of $\mathbf{F_a}(z)$ and $\mathbf{P_a}(z)$.

- *Coefficient Matrices*– The first one has a diamond structure,

$$\mathbf{F} := \begin{bmatrix} 0 & & d_0 & d_N & & 0 \\ & \iddots & & & \ddots & \\ d_{N/2-1} & & & & & d_{N+N/2-1} \\ d_{N/2} & & & & & d_{N+N/2} \\ & \ddots & & & \iddots & \\ 0 & & d_{N-1} & d_{2N-1} & & 0 \end{bmatrix}$$

The second one has a bi-diagonal structure,

$$\mathbf{C}_i := \begin{bmatrix} c_0^i & & & & & 1 \\ & \ddots & & & \iddots & \\ & & c_{N/2-1}^i & 1 & & \\ & & 1 & c_{N/2}^i & & \\ & \iddots & & & \ddots & \\ 1 & & & & & c_{N-1}^i \end{bmatrix}$$

with real or complex coefficients.

- *Standard Delay Matrix*– Increases the filter length and the system delay.

$$\mathbf{D}(z) := \begin{bmatrix} z^{-1} & & & & & \\ & \ddots & & & & \\ & & z^{-1} & & & 0 \\ 0 & & & 1 & & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix}$$

Its causal inverse needs a multiplication by z^{-1}.

$$z^{-1} \cdot \mathbf{D}^{-1}(z) = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & 0 \\ 0 & & & z^{-1} & & \\ & & & & \ddots & \\ & & & & & z^{-1} \end{bmatrix}$$

- *Zero-Delay Matrices*– Increase the filter length but not the system delay.

$$\mathbf{G}_i(z) := \begin{bmatrix} g_0^i z^{-1} & & & & & 1 \\ & \ddots & & & \cdots & \\ & & g_{N/2-1}^i z^{-1} & 1 & & \\ & & 1 & 0 & & \\ & \cdots & & & \ddots & \\ 1 & & & & & 0 \end{bmatrix}$$

Their inverse is causal.

$$\mathbf{G}_i^{-1}(z) = \begin{bmatrix} 0 & & & & & 1 \\ & \ddots & & & \cdots & \\ & & 0 & 1 & & \\ & & 1 & -g_{N/2-1}^i z^{-1} & & \\ & \cdots & & & \ddots & \\ 1 & & & & & -g_0^i z^{-1} \end{bmatrix}$$

$$\mathbf{E}_0 := \begin{bmatrix} 0 & & & & & e^0_N \\ & \ddots & & & \iddots & \\ & & 0 & e^0_{N+N/2-1} & & \\ & & e^0_{N+N/2} & e^0_{N/2}z^{-1} & & \\ & \iddots & & & \ddots & \\ e^0_{2N-1} & & & & & e^0_{N-1}z^{-1} \end{bmatrix}.$$

$$\mathbf{E}_i := \begin{bmatrix} 0 & & & & & 1 \\ & \ddots & & & \iddots & \\ & & 0 & 1 & & \\ & & 1 & e^i_{N/2}z^{-1} & & \\ & \iddots & & & \ddots & \\ 1 & & & & & e^i_{N-1}z^{-1} \end{bmatrix},$$

A cascade with low system delay

The following cascade yields a modulated filter bank

$$\mathbf{P_a} = \left(\prod_{i=1}^{m} \mathbf{C}_i \cdot \mathbf{D}^2(z)\right) \cdot \mathbf{F} \cdot \mathbf{D}(z) \cdot \left(\prod_{i=1}^{n} \mathbf{G}_i(z)\right) \mathbf{T}$$

The synthesis polyphase matrix for perfect reconstruction

$$\mathbf{P_s} = \mathbf{T}^{-1} \left(\prod_{i=0}^{n-1} \mathbf{G}_{n-i}^{-1}(z)\right) \cdot \mathbf{D}^{-1}(z) \cdot z^{-1} \cdot \mathbf{F}^{-1} \cdot \left(\prod_{i=0}^{m-1} \mathbf{D}^{-2}(z) \cdot z^{-2} \cdot \mathbf{C}_{m-i}^{-1}\right)$$

$$\text{Filter Length} = m2N + nN + 2N$$

$$\text{System Delay} = m2N + 2N - 1$$

$$\text{Number of Coefficients} = mN + 2N + nN/2$$

Orthogonal filter banks with standard system delay are a special case of the above formulation. They result if $\mathbf{F}$ and $\mathbf{C}_i$ are restricted to be orthogonal and no zero-delay matrices are used. This leads to the ELT of Malvar [4, 5].

A cascade with minimum system delay

$$\mathbf{P_a} = \prod_{i=0}^{m-1} \mathbf{E}_i \cdot \mathbf{T} \tag{2}$$

$$\mathbf{P_s} = \mathbf{T}^{-1} \cdot \prod_{i=0}^{m-1} \mathbf{E}_{m-1-i}^{-1} \tag{3}$$

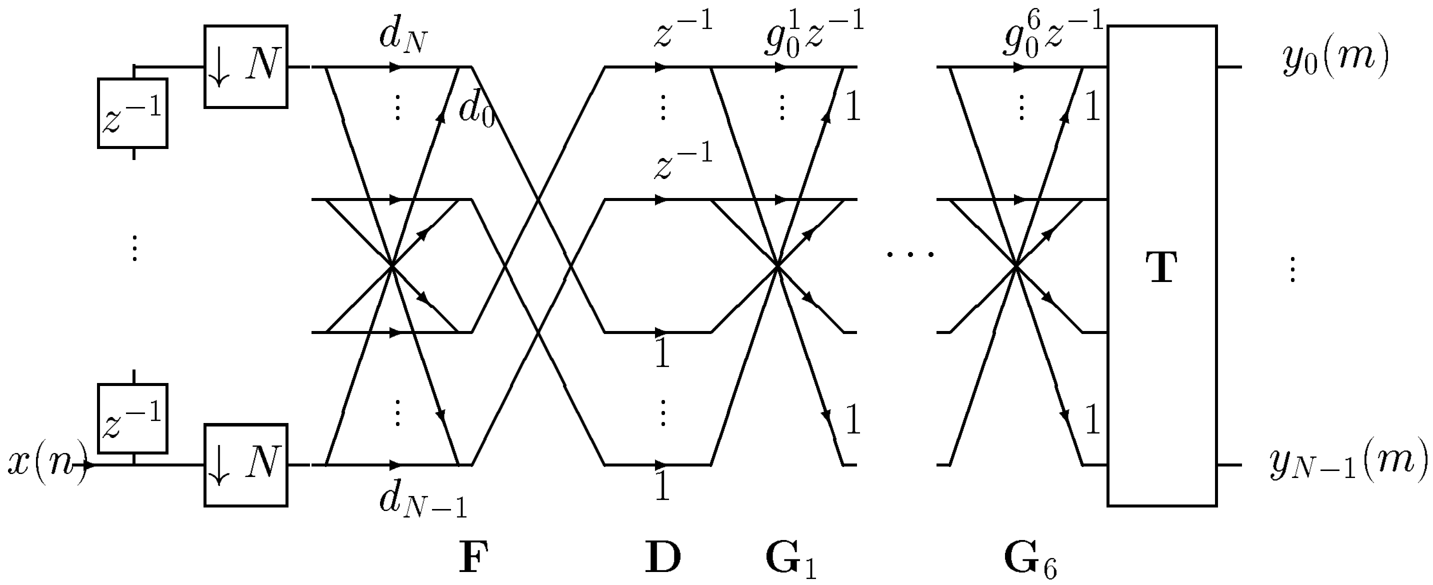
$$\text{Filter Length} = mN + 0.5N$$

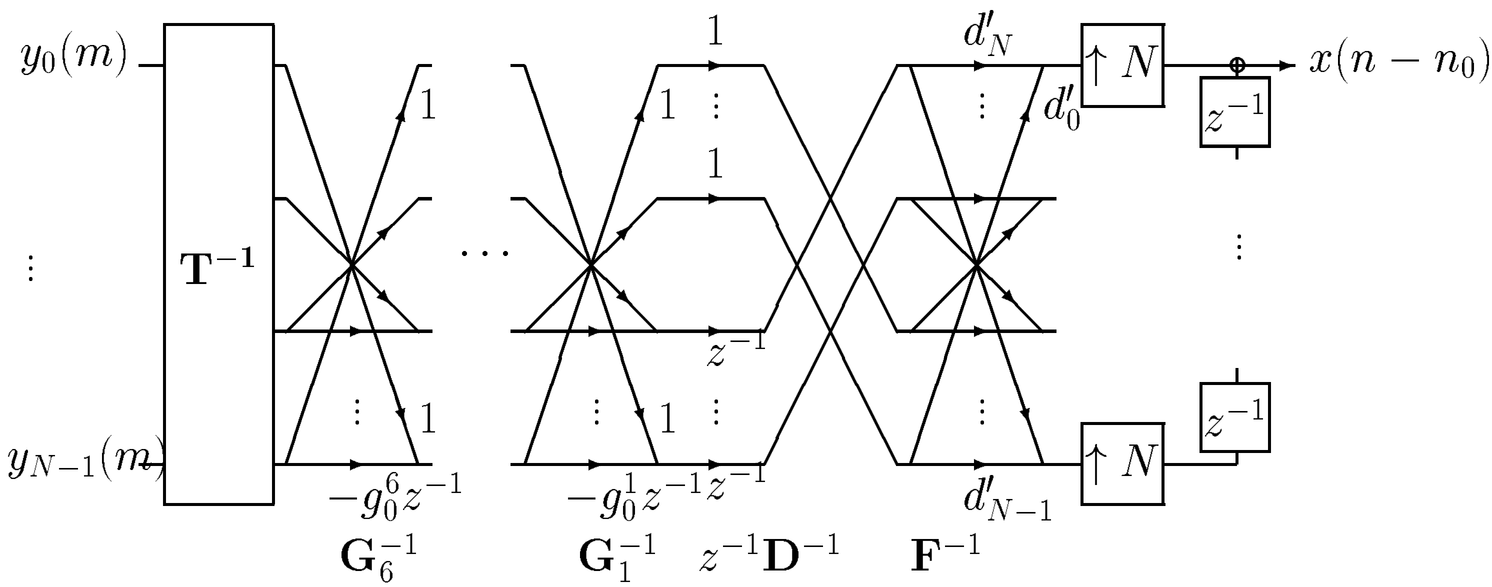
$$\text{System Delay} = N - 1$$

Optimization

- Is used to obtain the desired frequency responses
- Only the baseband filters need to be optimized
- An algorithm similar to the method of "conjugate directions" was used
- It was found to be relatively robust and fast
- Converges even for big filter banks

Structure for analysis (above) and synthesis (below),with $m = 0$ and $n = 6$

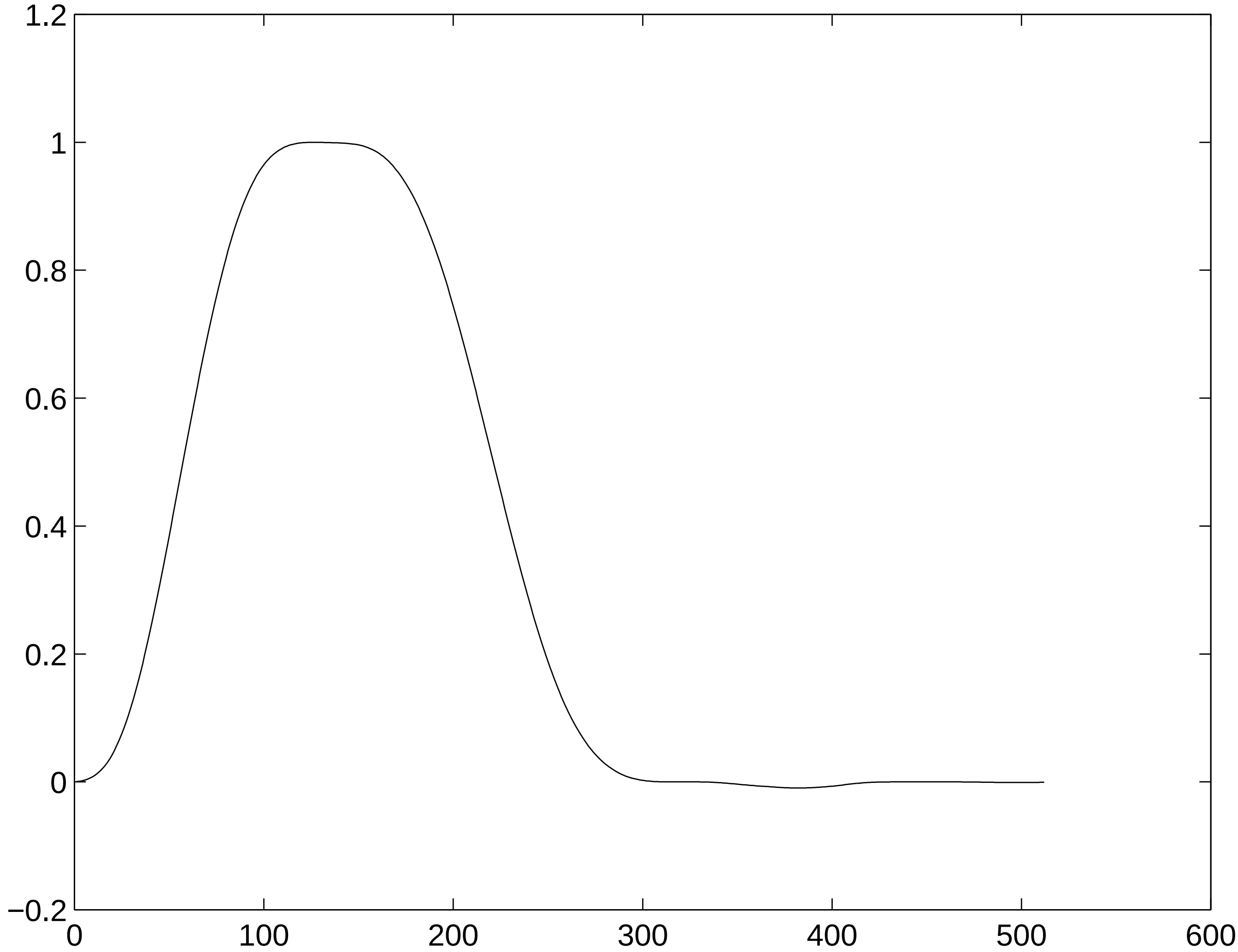




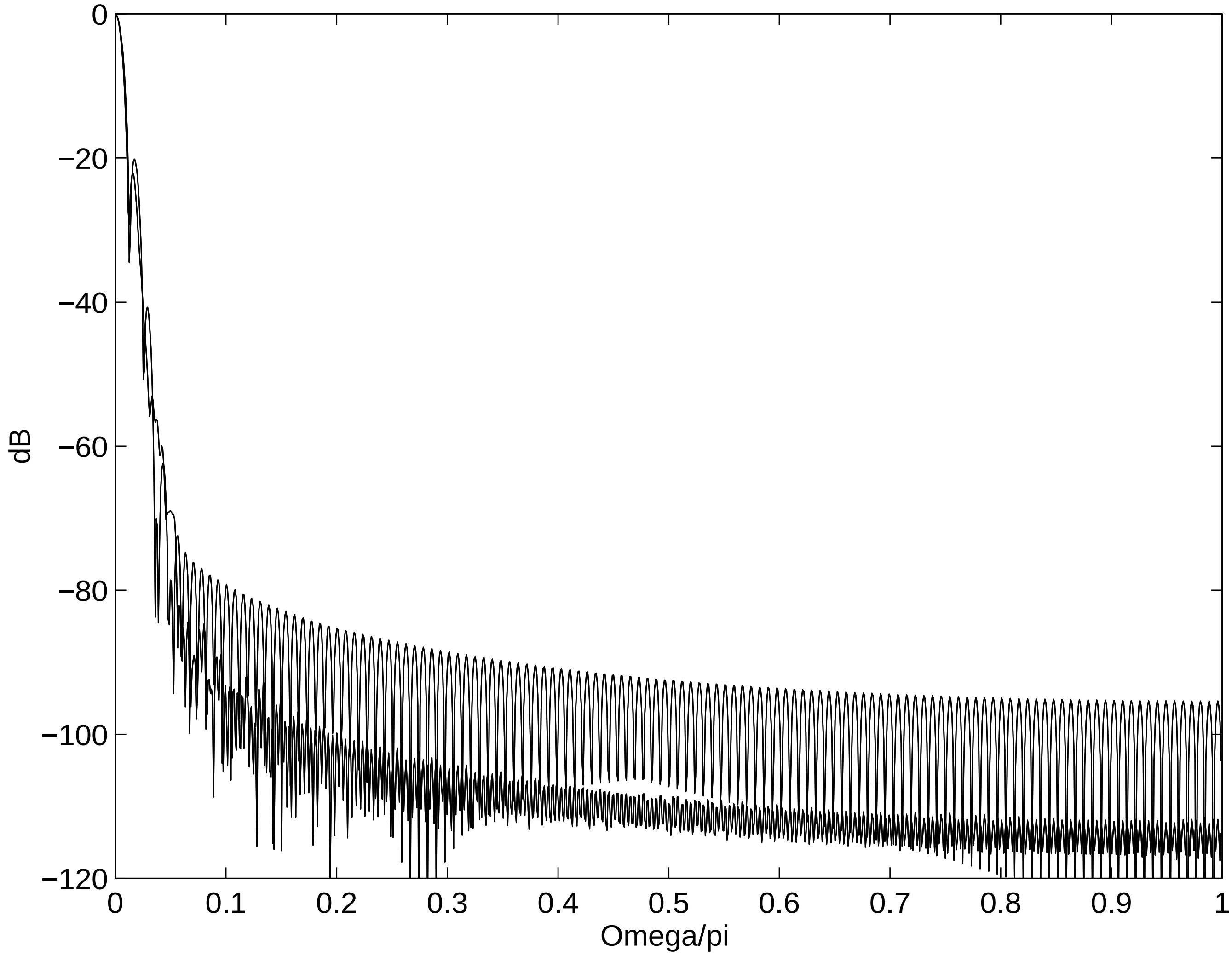
Properties

- Perfect reconstruction modulated filter bank
- Arbitrary filter length
- Realization with a fast algorithm
- Low system delay possible (independent of filter length)
- Optimization even for big filter banks
- Bi-orthogonal filter banks possible

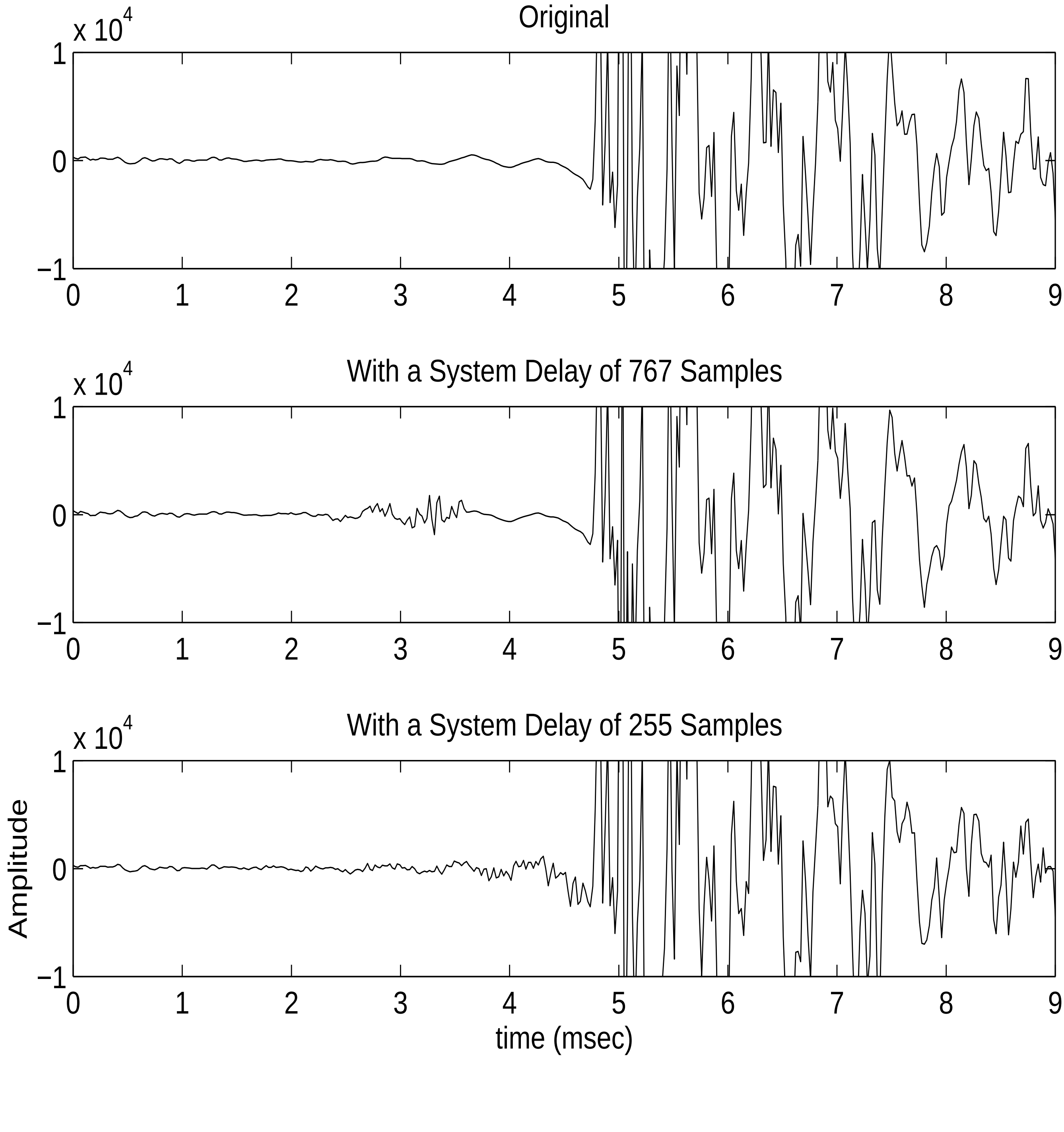
Example, baseband impulse response of a 128 band low delay filter bank, length 512 taps, delay 255 samples for analysis and synthesis



Magnitude response of the low delay filter bank compared to an orthogonal standard delay filter bank with the same delay



Audio coding application example, comparison of the pre-echoes, $f_s = 44.1\text{kHz}$



Conclusions

- Low delay filter banks are realizable even with perfect reconstruction
- And for big filter banks
- They can be useful for applications like audio coding
- Or in low delay coding applications

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